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Two-Dimensional DOA Estimation for Two Parallel Uniform Linear Arrays based on Sparse Representation

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Abstracts: In this paper, the issue of two-dimensional direction-of-arrival (DOA) estimation for two parallel uniform linear arrays (TPULA) is studied, and a simple method for the angle estimation based on sparse representation (SR) is proposed. Through the SR of the cross covariance vector, the proposed algorithm can achieve automatically paired two-dimensional DOA estimation using just one-dimensional dictionary. The algorithm requires neither eigenvalue-decompositions nor prior knowledge of the source numbers. Furthermore, it has better estimation performance than the improved propagator method (PM) and DOA-matrix method, especially with low signal-to-noise ratio (SNR). The simulation results verify the effectiveness of the proposed algorithm.

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1. Introduction

The problem of two dimensional (2D) direction-of-arrival (DOA) estimation is a key issue in array signal processing for its applications in many fields including radar, sonar, wireless communication and so on [1]-[5]. 2D DOA estimation with two parallel uniform linear arrays (TPULA) has been studied by lots of researchers for its special array structure [6]-[7], [9]-[11]. A DOA-matrix method was proposed in [6], which can obtain the DOA estimations without additional pairing match. A polynomial root-finding method for DOA estimation with TPULA was proposed in [7], which can exploit the full array aperture. Subspace-based methods can obtain high-resolution angle estimations, but the requirement of the eigenvalue decomposition (EVD) of the covariance matrix makes them hard to implement. Propagator method (PM) [8] is well-known for it is free of EVD of the covariance matrix, so a fast DOA estimation algorithm based on PM for TPULA was proposed in [9]. Ref.[10] presented a more computationally efficient DOA estimation method via the shifts of the arrays, but it has worse angle estimation performance. An improved PM-based method was proposed in [11], which has improved estimation performance than the method in [9], but the estimation performance degrades quickly when the signal-to-noise ratio (SNR) is low.

Sparse representation (SR) based method for DOA estimation has attracted lots of attention for its super-resolution property and it is easy to implement. The most successful ℓ_1-SVD method [12] employs singular value decomposition (SVD) to concentrate the signal power and turns the angle estimation problem into a sparse recovery via multiple measurement vectors (MMV). The methods in [13]-[14] utilize the vectorization of the covariance matrix as a single measurement vector (SMV) to reduce the recovery complexity. However, the methods above are all for one-dimensional DOA estimation. The 2D DOA estimation will require the dictionary for two-dimensional angle, which will add high computational burden and perhaps will make the recovery method fail.

In this paper, a SR based algorithm for 2D DOA estimation with TPULA is proposed. The proposed algorithm has the following advantages: 1) it can achieve automatically paired two-dimensional estimations of angles, 2) it uses just one-dimensional dictionary, 3) it requires neither EVD nor prior knowledge of the source numbers, 4) it has better DOA estimation performance than the improved PM [11] and DOA-matrix method [6], especially with low SNR.

The remainder of this paper is organized as follows. Section 2 develops the data model for TPULA, and Section 3 presents the SR based algorithm for 2D DOA estimation and Cramer-Rao bound (CRB). The simulation results are presented in section 4 to verify the effectiveness of the proposed algorithm, while the conclusions are made in Section 5.

Notation: $(.)^T$ 、 $(.)^H$ 、 $(.)^{-1}$ and $(.)^+$ denote transpose, conjugate-transpose, inverse, pseudo-inverse operations, respectively. diag(**v**) stands for diagonal matrix whose diagonal element is a vector **v**. $\mathbf{I}_M$ and $\mathbf{\Pi}_M$ are $M \times M$ identity matrix and reverse identity matrix, respectively. $\odot$, $\otimes$ and $\circ$ are the Hadamard product, Kronecker product and Khatri_rao product, respectively. $\|\cdot\|_1$ and $\|\cdot\|_2$ are l_1 and l_2 norm, respectively. $trace(.)$ means the trace of a matrix, $E[.]$ is expectation operator and $angle(\cdot)$ is to get the phase. $vec(\cdot)$ is the vectorization operation.

2. Data model

As Fig.1 shows, the TPULA is located in the x-y plane with half-wavelength being the inter-element spacing, and it can be divided into two M-element subarrays. Assume that there are K far-field narrowband sources impinging on the arrays with the DOA being $(\alpha_k, \beta_k), k = 1,...,K$, among which α_k is the angle between the wave line and Y-axis and β_k is the angle between the wave line and X-axis. The outputs of the two subarrays can be expressed as

$$\mathbf{x}_1(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}_1(t) \tag{1}$$

$$\mathbf{x}_2(t) = \mathbf{A}\mathbf{\Phi}\mathbf{s}(t) + \mathbf{n}_2(t) \tag{2}$$

where $\mathbf{A} = [\mathbf{a}(\alpha_1), \cdots, \mathbf{a}(\alpha_K)]$ denotes the direction matrix of subarray 1, and $\mathbf{a}(\alpha_k) = [1, e^{-j\pi\cos\alpha_k}, \cdots, e^{-j\pi(M-1)\cos\alpha_k}]^T, k = 1,...,K$. $\mathbf{\Phi} = diag(e^{-j\pi\cos\beta_1}, \cdots, e^{-j\pi\cos\beta_K})$, and $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T$ is the signal vector with $s_k(t)$ being the kth source. $\mathbf{n}_1(t)$ and $\mathbf{n}_2(t)$ are additive white Gaussian noise vectors with zeros mean and covariance matrix $\sigma^2\mathbf{I}_M$, and they are assumed to be independent to each other and uncorrelated with the signals.

3. 2D DOA estimation for TPULA

3.1 2D DOA estimation algorithm based on SR

The cross covariance matrix of the two outputs is

$$\mathbf{R}_{12} = E[\mathbf{x}_2(t)\mathbf{x}_1^H(t)] \tag{3}$$

With the assumption above [illegible] substitute Eqs.(1)-(2) into Eq.(3), the cross covariance matrix can be expressed as

$$\mathbf{R}_{12} = \mathbf{A\Phi R}_s\mathbf{A} \tag{4}$$

where $\mathbf{R}_s = E[\mathbf{s}(t)\mathbf{s}^H(t)] = diag(\sigma_1^2,...,\sigma_K^2)$, and $\sigma_k^2, k=1,...,K$ represents the signal power.

Utilize the vectorization operation, the cross covariance vector is

$$\begin{aligned}\mathbf{r}_{12} &= vec(\mathbf{R}_{12}) \\ &= vec(\mathbf{A\Phi R}_s\mathbf{A}) \\ &= (\mathbf{A}^* \circ \mathbf{A})\boldsymbol{\eta}\end{aligned} \tag{5}$$

where $\boldsymbol{\eta}=[e^{-j\pi\cos\beta_1}\sigma_1^2,\cdots,e^{-j\pi\cos\beta_K}\sigma_K^2]$. The kth column of $\mathbf{A}^* \circ \mathbf{A}$ is $\mathbf{a}(\alpha_k)^* \otimes \mathbf{a}(\alpha_k) \in \mathbb{C}^{M^2\times 1}$, which has only 2$M$-1 different items and its long length will add the complexity of the sparse recovery next. So a reduced-dimension transformation should be considered first. Introduce a vector $\mathbf{b}(\alpha_k) \in \mathbb{C}^{(2M-1)\times 1}$, which contains the 2$M$-1 different items in $\mathbf{a}(\alpha_k)^* \otimes \mathbf{a}(\alpha_k)$

$$\mathbf{b}(\alpha_k) = [e^{j(M-1)\pi\cos\alpha_k},...,e^{j\pi\cos\alpha_k},1,e^{-j\pi\cos\alpha_k},\cdots,e^{-j(M-1)\pi\cos\alpha_k}]^T \tag{6}$$

It can be derived that

$$\mathbf{a}(\alpha_k)^* \otimes \mathbf{a}(\alpha_k) = \mathbf{Gb}(\alpha_k) \tag{7}$$

where $\mathbf{G}=[\mathbf{G}_1^T,...,\mathbf{G}_M^T]^T$ and $\mathbf{G}_m=[\mathbf{0}_{M\times(m-1)},\mathbf{\Pi}_M,\mathbf{0}_{M\times(M-m)}], m=1,...,M$. Use the orthogonal form of $\mathbf{G}^H$ as the reduced-dimension transformation matrix to avoid the noise power gain, then the data in (5) becomes

$$\begin{aligned}\mathbf{r} &= \underbrace{(\mathbf{G}^H\mathbf{G})^{-\frac{1}{2}}\mathbf{G}^H}_{\mathbf{C}}\mathbf{r}_{12} \\ &= \mathbf{W}\mathbf{B}\boldsymbol{\eta}\end{aligned} \tag{8}$$

where $\mathbf{W}=(\mathbf{G}^H\mathbf{G})^{\frac{1}{2}}$ is a diagonal matrix, and $\mathbf{B}=[\mathbf{b}(\alpha_1),...,\mathbf{b}(\alpha_K)]$. The length of the data are reduced from M^2 to 2M-1 via (8), and three advantages can be shown when compared to the original data in (1)-(2). 1, There is almost no array aperture loss, 2, the noise influence is reduced, 3, use only SMV to reduce the complexity in the sparse recovery next.

Now the sparse representation can be employed for the angle estimation. Let $\tilde{\alpha}_1,\tilde{\alpha}_2,\cdots,\tilde{\alpha}_L (L \gg K)$ be a sampling grid of all angles of interest, which obviously contains the true angles $\alpha_1,\alpha_2,\cdots,\alpha_K$. Construct a fat matrix $\mathbf{\Theta}=[\mathbf{b}(\tilde{\alpha}_1),...,\mathbf{b}(\tilde{\alpha}_L)]$, which is called the dictionary, and it can be indicated that $\mathbf{\Theta}$ contains all the columns of $\mathbf{B}$. Correspondingly, expand $\boldsymbol{\eta}$ to a taller vector $\boldsymbol{\rho}\in\mathbb{C}^{L\times 1}$, whose elements corresponding to the true angles keep the same with those in $\boldsymbol{\eta}$, and the others are all-zero. It can be shown that $\mathbf{r}=\mathbf{W}\mathbf{B}\boldsymbol{\eta}=\mathbf{W}\mathbf{\Theta}\boldsymbol{\rho}$, with $\boldsymbol{\rho}$ being a K-sparse vector.

Employ the sparse recovery technique positions and values of non-zero elements in $\boldsymbol{\rho}$ will give the estimations of α_k and β_k, respectively. But in practice, the cross covariance matrix in (3) can only be estimated via finite samples, that is $\hat{\mathbf{R}}_{12}=\frac{1}{N}\sum_{t=1}^{N}\mathbf{x}_2(t)\mathbf{x}_1^H(t)$, which will cause the estimate error even with no noise, and that will influence the sparse recovery performance. To enhance the robustness the algorithm, the sparse recovery problem should be formulated as

$$\begin{aligned}&\min \|\hat{\boldsymbol{\rho}}\|_1 \\ &s.t. \|\hat{\mathbf{r}}-\mathbf{W}\mathbf{\Theta}\hat{\boldsymbol{\rho}}\|_2 \le \xi\end{aligned} \tag{9}$$

where ξ is the up bound for the fitting error, whose selection will be detailed derived next.

The mth column of $\hat{\mathbf{R}}_{12}$ is

$$\hat{\mathbf{r}}_{12m} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}_2(t)x_{1m}^{*}(t) \tag{10}$$

where $x_{1m}(t)$ is the mth element of $\mathbf{x}_1(t)$. According to [15], the signals are independent from sample to sample and circularly Gaussian distributed. Then it can be derived that

$$\begin{aligned} E[\hat{\mathbf{r}}_{12m}\hat{\mathbf{r}}_{12n}{}^{H}] &= \frac{1}{N^2}\sum_{t=1}^{N}\sum_{p=1}^{N}E[\mathbf{x}_2(t)x_{1m}^{*}(t)\mathbf{x}_2^{H}(p)x_{1n}(p)] \\ &= \mathbf{r}_{12m}\mathbf{r}_{12n}{}^{H} + \frac{1}{N}\mathbf{R}_{1nm}\mathbf{R}_2 \end{aligned} \tag{11}$$

where $\mathbf{R}_{1nm}$ is the (n,m)th element of covariance matrix $\mathbf{R}_1 = E[\mathbf{x}_1(t)\mathbf{x}_1^{H}(t)]$ and $\mathbf{R}_2 = E[\mathbf{x}_2(t)\mathbf{x}_2^{H}(t)]$. From Eq.(11), it can be shown that the mean square error of $\mathbf{r}_{12}$ is

$$E[(\hat{\mathbf{r}}_{12}-\mathbf{r}_{12})(\hat{\mathbf{r}}_{12}-\mathbf{r}_{12})^{H}] = \frac{1}{N}(\mathbf{R}_1^{T}\otimes\mathbf{R}_2) \tag{12}$$

Then the mean square error of the reduced-dimension data $\mathbf{r}$ will be

$$\begin{aligned} E[(\hat{\mathbf{r}}-\mathbf{r})(\hat{\mathbf{r}}-\mathbf{r})^{H}] &= \mathbf{C}E[(\hat{\mathbf{r}}_{12}-\mathbf{r}_{12})(\hat{\mathbf{r}}_{12}-\mathbf{r}_{12})^{H}]\mathbf{C}^{H} \\ &= \frac{1}{N}\mathbf{C}(\mathbf{R}_1^{T}\otimes\mathbf{R}_2)\mathbf{C}^{H} \end{aligned} \tag{13}$$

When the samples are sufficiently large, use $\hat{\mathbf{R}}_1 = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}_1(t)\mathbf{x}_1^{H}(t)$ and $\hat{\mathbf{R}}_2 = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}_2(t)\mathbf{x}_2^{H}(t)$ to approximately replace $\mathbf{R}_1$ and $\mathbf{R}_2$ in Eq.(13), then the error bound ξ in Eq.(9) can be set as

$$\xi = \sqrt{trace(\frac{1}{N}\mathbf{C}(\hat{\mathbf{R}}_1^{T}\otimes\hat{\mathbf{R}}_2)\mathbf{C}^{H})} \tag{14}$$

Finally, the fitting problem in Eq.(9) can solved by the sparse recovery tool: SPGL1 [16]. The positions of the non-zeros elements in the recovery sparse vector $\hat{\boldsymbol{\rho}}$ will give the estimations of the angles $\alpha_k, k=1,\ldots,K$. Furthermore, the non-zero elements themselves will give the estimations of $\boldsymbol{\eta}$, and the angles β_k can be estimated via

$$\hat{\beta}_k = \arccos(-angle(\hat{\boldsymbol{\eta}}(k))/\pi) \tag{15}$$

where $\hat{\boldsymbol{\eta}}(k)$ is the *k*th element of the estimated vector $\hat{\boldsymbol{\eta}}$.

As α_k and β_k are estimated corresponding to the same vector $\hat{\boldsymbol{\rho}}$, they are automatically paired. Furthermore, the sparse recovery in (9) requires no prior knowledge of the source number, which is needed in improved PM [11] and DOA-matrix method [6].

Till now, we have achieved the proposal for the SR based algorithm for 2D DOA estimation with TPULA. The major steps are shown as follows:

1) Estimate the cross covariance matrix via $\hat{\mathbf{R}}_{12} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{x}_2(t)\mathbf{x}_1^H(t)$.

2) Utilize the vectorization operation and reduced-dimension transformation on $\hat{\mathbf{R}}_{12}$ to obtain the SMV $\hat{\mathbf{r}}$.

3) Construct the one-dimensional dictionary $\boldsymbol{\Theta}$, calculate the error bound ξ via (14), and substitute them as well as $\hat{\mathbf{r}}$ into the sparse recovery tool to estimate the sparse vector.

4) Obtain the estimations of α_k and β_k by exploiting the positions and values of non-zero elements in the recovery sparse vector.

3.2 Cramer-rao Bound (CRB)

CRB is the low bound for angle estimation error based on the data model, and it can be used to measure the performance of the algorithms. Let $\boldsymbol{\Omega}=\begin{bmatrix}\mathbf{A}\\ \mathbf{A}\boldsymbol{\Phi}\end{bmatrix}$, according to [17], the CRB for the 2D DOA estimation with TPULA can be derived as

$$CRB = \frac{\sigma^2}{2N}\left\{\mathrm{Re}\left[\mathbf{D}^H\boldsymbol{\Pi}_{\boldsymbol{\Omega}}^{\perp}\mathbf{D}\odot\hat{\mathbf{P}}_w^{\ T}\right]\right\}^{-1} \tag{16}$$

where $\mathbf{D} = \left[\frac{\partial\mathbf{a}_1}{\partial\alpha_1},\ldots,\frac{\partial\mathbf{a}_K}{\partial\alpha_K},\frac{\partial\mathbf{a}_1}{\partial\beta_1},\ldots,\frac{\partial\mathbf{a}_K}{\partial\beta_K}\right]$ with $\mathbf{a}_k$ being the *k*th column of $\boldsymbol{\Omega}$. $\hat{\mathbf{P}}_w = \begin{bmatrix}\hat{\mathbf{P}}_s & \hat{\mathbf{P}}_s\\ \hat{\mathbf{P}}_s & \hat{\mathbf{P}}_s\end{bmatrix}$,

$$\hat{\mathbf{P}}_s = \frac{1}{N}\sum_{t=1}^{N}\mathbf{s}(t)\mathbf{s}^H(t); \quad \mathbf{\Pi}_{\mathbf{\Omega}}^{\perp} = \mathbf{I}_{2M} - \mathbf{\Omega}\left(\mathbf{\Omega}^H\mathbf{\Omega}\right)^{-1}\mathbf{\Omega}^H .$$

4. Simulation results

Define root mean square error (RMSE) of the angle estimation as

$$RMSE = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{D}\sum_{d=1}^{D}[(\hat{\alpha}_{k,d}-\alpha_k)^2+(\hat{\beta}_{k,d}-\beta_k)^2]} \tag{17}$$

where $\hat{\alpha}_{k,d}$ and $\hat{\beta}_{k,d}$ are the estimations of α_k and β_k of the dth Monte Carlo trial, respectively. Assume that there are $K = 3$ uncorrelated sources with DOA being $(\alpha_1,\beta_1)=(50^\circ,55^\circ)$, $(\alpha_2,\beta_2)=(65^\circ,40^\circ)$ and $(\alpha_3,\beta_3)=(80^\circ,75^\circ)$. M=8 and N=500 samples are collected, and D=500 trials are presented.

Fig. 2 depicts the angle estimation results of the algorithm with SNR=5dB. It is shown that the 2D DOA can be accurately estimated.

The angle estimation performance comparison between the proposed algorithm, improved PM, DOA-matrix method and CRB versus SNR and sample number are presented in Fig.3 and Fig.4, respectively. From Fig3, it can be indicated that the angle estimation performance of the proposed algorithm is better than the other two methods, especially with low SNR. Meanwhile, it can be shown from Fig.4 that the proposed algorithm still outperforms the other two methods with different sample numbers.

The angle merger problem often occurs for source location, and when the sources have the same angle α, all the three methods will fail. But when the sources have the same angle β, the proposed algorithm can work well, while the other two will still fail. The estimation results when two sources have the same angle β are shown in Fig.5, which can demonstrate the opinion above.

5. Conclusion

In this paper, a SR based 2D DOA estimation algorithm for TPULA has been proposed. Through the SR of the cross covariance vector via one-dimensional dictionary, the algorithm can obtain 2D angle estimation with automatically pairing match. The algorithm requires neither EVD of the covariance matrix nor prior information of the source number. Furthermore, it has better estimation performance than the improved PM and DOA-matrix method, especially with low SNR.

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Figure
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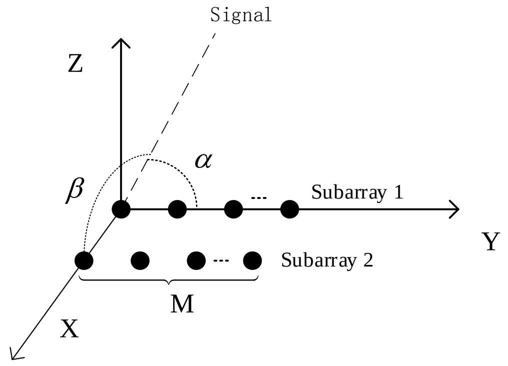

Fig.1 Illustration of the array geometry

Figure

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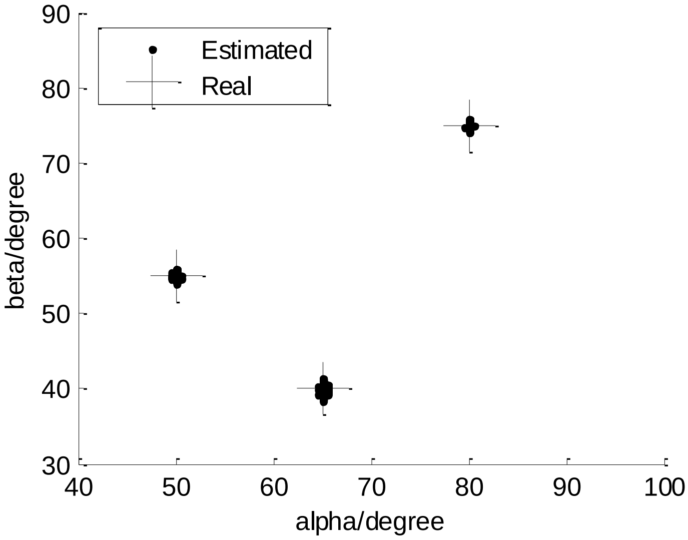

Fig.2 Angle estimation results of the proposed algorithm

Figure

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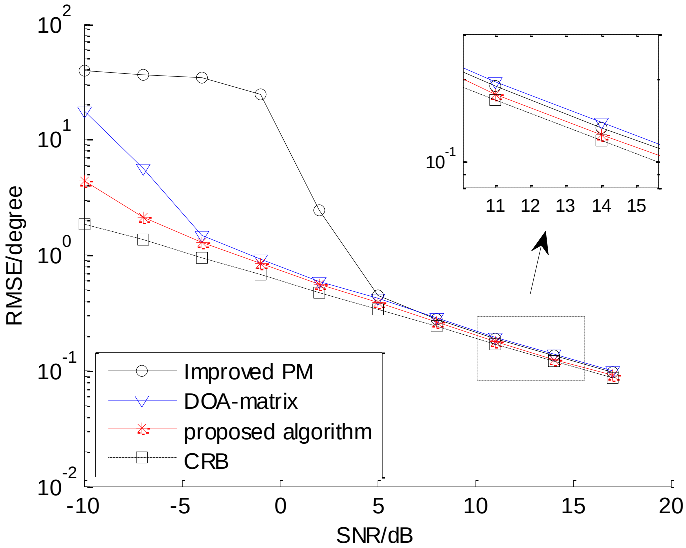

Fig.3 Angle estimation performance comparison versus SNR

Figure
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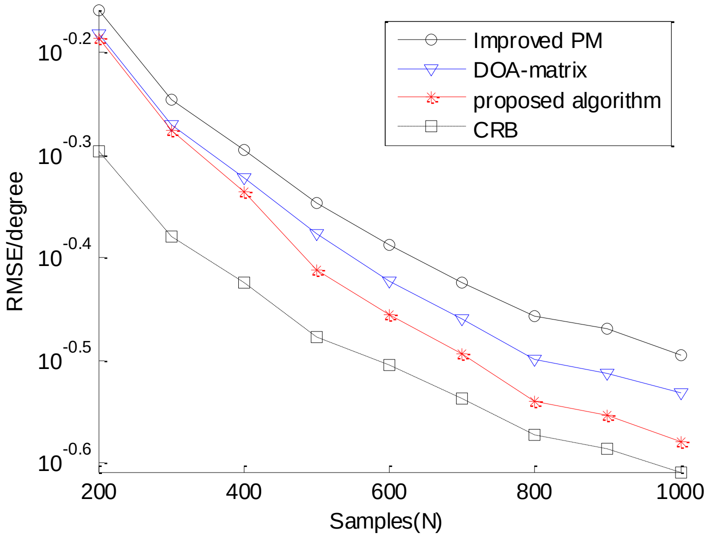

Fig.4 Angle estimation performance comparison versus sample number (SNR=5dB)

Figure

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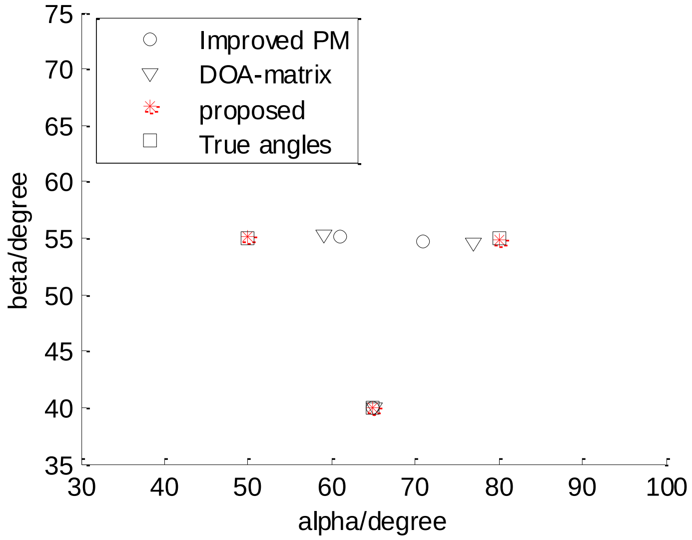

Fig.5 Angle estimation results with the same **Error! Objects cannot be created from editing field codes.**